Digital Signal Processing

Report 2

Solution of the report.

request: classify the given systems according to Linearity, Stability, Causality, Dynamicity and shift invariance.

a) $y(n) = \sum_{k=-a}^{n+1} X(k)$

* Linearity:

$X_1(k) \longrightarrow y(k) = \sum_{k=-a}^{n+1} X_1(k)$

$X_2(k) \longrightarrow y_2 = \sum_{k=-a}^{n+1} X(k)$

$(X_1(k) + X_2(k)) \longrightarrow y_3 = \sum_{k=-a}^{n+1} X_1(k) + X_2(k) = \sum_{k=-a}^{n+1} X_1(k) + \sum_{k=-a}^{n+1} X_2(k)$

$\therefore y_3 = y_1 + y_2 \longrightarrow$ Linear System

* Stability:

for Bounded input $X(k)$

$y(n) = \sum_{k=-a}^{n+1} X(k) = X(-a) + X(-a+1) \dots + X(0) + X(1) + \dots + X(n) + X(n+1)$

if $a = \infty$ then $\sum_{k=-a}^{n+1} X(k) \longrightarrow \infty$ Unbounded Output

$\therefore$ System is Unstable

* Causality:

$y(n) = \sum_{k=-a}^{n+1} X(k) = X(-a) + \dots + X(0) + \dots + X(n) + X(n+1)$

the output $y(n)$ depends on future Sample $X(n+1)$

$\therefore$ System is non-Causal

* Dynamicity:

$y(n)$ depends on $x(n+1)$ and previous Samples $x(-a)$ --- $x(n-1)$

$\therefore$ System is <u>Dynamic</u>

* Time (Shift) invariance

$$\left.\begin{aligned} y(n,b) &= \sum_{k=-a}^{n+1} x(k-b) \\ y(n-k) &= \sum_{k=-a}^{n+1} x(k-b) \end{aligned}\right\}$$

The Output is linear Summation of inputs.

$$\left.\begin{aligned} y(n,b) &= \sum_{k=-a}^{n-b+1} x(k) \\ y(n-b) &= \sum_{k=-a}^{n-b+1} x(k) \end{aligned}\right\} \text{the Same}$$

$\therefore$ System is <u>shift Invarent</u>

b) $y(n) = x(2n)$

* linearity:

$x_1(n) \Longrightarrow y_1 = x_1(2n)$

$x_2(n) \longrightarrow y_2 = x_2(2n)$

$(x_1(n) + x_2(n)) \longrightarrow y_3(n) = x_1(2n) + x_2(2n) = y_1 + y_2$

System is <u>linear</u>

* Stability:

For $x(n)$: bounded $\longrightarrow y(n) = x(2n)$ Bounded

System is <u>stable</u>

* Causality:

$y(n)$ depends on Sample at $2n$ (future)

System is <u>non-Causal</u>

* Dynamicity:

$y(n)$ doesn't depends only on Current input

System is <u>dynamic</u>

* Shift Invarience:

$$\left.\begin{aligned} y(n,k) &= x(2n-k) \\ y(n-k) &= x(2n-2k) \end{aligned}\right\} y(n,k) \neq y(n-k)$$

System is <u>shift varient</u>

c) $y = X(-n)$

* Linearity:

$X_1(n) \longrightarrow y_1 = X_1(-n)$

$X_2(n) \longrightarrow y_2 = X_2(-n)$

$(X_1(n) + X_2(n)) \longrightarrow y_3 = X_1(-n) + X_2(-n) = y_1 + y_2$

System is <u>Linear</u>

* Causality:

at $n = -1 \longrightarrow y(-1) = X(-n) = X(1)$

Output depends on future Sample

System is <u>non Causal</u>

* Stability:

For Bounded Input $X(n)$, The Output is bounded $y = X(-n)$

System is <u>Stable</u>

* Dynamicity:

response doesn't depend on current input only

System is <u>dynamic</u>

* Time invarience:

$$\left.\begin{aligned} y(n,k) &= X(-n-k) \\ y(n-k) &= X(-(n-k)) = X(-n+k) \end{aligned}\right\} \; y(n,k) \neq y(n-k)$$

System is <u>Shift-Variant</u>